BRNO UNIVERSITY OF TECHNOLOGY FACULTY OF INFORMATION TECHNOLOGY

Excel@FIT 2024
http://excel.fit.vutbr.cz

Oblique Elliptical Basis Function (OEBF)

Leoš Nevoral

Abstract
This paper presents a novel approach to implementing an oblique elliptical radial basis function (OEBF) within neurons. Unlike previous methodologies that rely on the center function of the ellipse [1, 2, 3], the proposed solution leverages the definition of ellipses based on their focal points. By utilizing the euclidean distance of points, multidimensional data is projected into a 2D space akin to RBF neurons. [4, chapter 6] However, instead of constructing circles, we employ focal points to form ellipses, thus partitioning the space into elliptical areas, which offer greater flexibility compared to radial areas. This innovative approach enhances the design of radial basis functions while preserving their essential characteristics. Our method demonstrates up to **9.05 %** improvement in the accuracy from RBF neurons and improvement in reduction of the required number of neurons by up to **45.27 %**. Furthermore, compared to conventional elliptical solutions, it minimizes the number of weights managed by each neuron and yields competitive results.

*xnevor03@stud.fit.vutbr.cz, *Faculty of Information Technology, Brno University of Technology*

1. Introduction

[Current state] The field of elliptical basis function has an undiscovered potential. Most work done in this field focuses on creating ellipsoids, that are represented by the central function of ellipse. [1, 2, 3]

$$u = \sum_{i=0}^{i=n} \frac{(C_i + X_i)^2}{w_i^2}$$

These solutions have often proven to be greater than classic RBF. But it differs greatly from the simplicity of RBF and the neurons have a lot more number of weights to manage, resulting in the fact that some of the weights are never needed to be adjusted during the learning process, pointing to a fact, that simpler shape might be more fitting.

[Proposed solution] To achieve above mentioned shape, it's essential to find a way to simplify the n-dimensional space, which represents data. The proposed solution is inspired by the methods, the RBF neurons use. Instead of using the central function, its focus is on using the focal points of ellipse and it's semi-axis lengths to create the elliptical area, similarly to RBF function, which uses it's center and weight to create radial areas. This way the amount of weights each neuron has would greatly reduce from n to 2 in comparison to EBF neurons and flexibility of this function would be enhanced from RBF, whilst keeping it's core idea.

[Results] To measure the abilities of each neuron a Restricted Coulomb Energy (RCE) neural network is employed. Main benefit of the learning algorithm used by RCE is the 100 % guarantied accuracy on training dataset after learning. The neurons used in the network differ by the used basis function, but all neurons use step activation function, which activates when the vector, lies in boundaries of it's basis function. Learning method actions are as follows:

- It Applies a training vector too all neurons in hidden layer
- When a training vector is misclassified by a neuron, it reduces area of the neuron.
- If a training vector remains unclassified, it adds a new neuron, utilizing the vector as its center, or in our approach, as a focal point.
- After finishing every epoch, it checks if any changes were made to the hidden layer and if so it restarts the learning process.
- If no changes were made to the network it signifies, that the whole training dataset was classified correctly and the training process is finished.

This learning algorithm enables the establishment of a metric for determining the number of neurons required to cover a given dataset.

2. Solution

This basis function is defined as the euclidean distance of focal points from the input vector Equation 1. Then the neuron uses 2 weights representing the main a secondary semi-axis of ellipse. As activation function it uses the Activation function, which is a step activation function, that is activated if u is less than 2 lengths of main axis, otherwise it remains inactivated. This way the solution compares to RBF, with the difference of using 2 points and weights instead of one. The proposed simpler representation based on the focal points of ellipses also has it's problems to solve. The first problem is finding the best focal points. In order to find a somewhat great focal points, the following algorithm is used.

- Select unclassified vector from training dataset.
- Pair it with a second unclassified vector from training dataset.
- If both vectors and center between them is unclassified, select them as focal points and center of new ellipse.

Another problem is manipulation of the ellipse semi-axis. To accomplish that, we need to move the focal point inside the multidimensional space along the main axis, using Equation 3 we can calculate new focal points position during shrinking. This underlies the first unsolved bigger problem and that is changing main axis with secondary one. To do that the focal points would have to move in direction of second semi-axis which isn't yet possible with our current solution. To even out the problem, any time the ellipse would need to create a smaller main axis, than secondary semi-axis, the secondary semi-axis is reduced to the same length, essentially creating a RBF shaped neuron. But a strength that this solution introduces as byproduct is the removal of rotation matrices to rotate the ellipse. The rotation is naturally encoded in the positions of focal points.

3. Testing

To implement the solution C++ is used. Each tested neuron has it's own class, which implements the methods specified in common interface. The common interface consists of methods "contain", "shrink" and "getLabel". Which are all essential for Restricted Coulomb Energy Neural Network, that is used for the testing.

3.1 OEBF

To complete the common interface, we need an algorithm to optimize the shrinking of neuron. That is done by a binary search algorithm that finds approximation on interval $< current\_length\_of\_axis, 0 >$, that is withing 10 % error margin from the optimal solution.

3.2 Other neuron types

Compared neurons are implemented using the standard algorithms. The EBF implementation is inspired by [1]. The RBF neuron is typically used in RCE networks and follows the description from [5].

3.3 Results

The Chart 1 shows the performance of all neuron types on the MNIST dataset. From the data we can determine a **6.77 %** improvement in the accuracy from RBF neurons and reduction of the required number of neurons by **45.27 %**. But compared to classic EBF solution we can see a slight drop in accuracy by **1.89 %** and growth in neuron count by **31.26 %**. Then on the Chart 2 performance on MNIST Fashion dataset is shown. Similar result are reached with our solution improving over RBF by **9.05 %** a reducing the number of neurons by **24.36 %** and compared to EBF it reaches **0.25 %** improvement in accuracy as well as **2.35 %** reduction of neurons. Unfortunately this data is not from testing on whole dataset, but with comparison to first result, it seems to follow similar curve.

4. Conclusion

Under the specified conditions, the proposed solution consistently yields more accurate predictions compared to the classic RBF approach, thus representing an improvement. While it approaches the performance of the traditional EBF solution, further refinement of the focal points selection algorithm or the development of an algorithm to adjust the main and secondary axes could potentially push the solution to surpass the capabilities of the classic EBF solution. This suggests a need for enhancing the effectiveness of our solution in future iterations.

Acknowledgements

I would like to thank my supervisor doc. Ing. František Zbořil, CSc. for their help and encouragement to participate in this event.

References

[1] M.C. Moed and C.-P. Lee. Design of an elliptical neural network with application to degraded character classification. In *IEEE International Conference on Neural Networks*, pages 1576–1582 vol.3, 1993.

[2] Tae Hong Park and Perry Cook. Radial/elliptical basis function neural networks for timbre classification. In *Journées d'Informatique Musicale 2005*, Saint-Denis, France, June 2005. Association Française d'Informatique Musicale and Centre de recherche en Informatique et Création Musicale.

[3] Hung-Wen Peng, Shie-Jue Lee, and Chie-Hong Lee. An oblique elliptical basis function network approach for supervised learning applications. *Applied Soft Computing*, 60:552–563, 2017.

[4] David Kriesel. *A Brief Introduction to Neural Networks*. 2007.

[5] František Zbořil. Neuronové sítě rbf a rce. materiály k přednáškám předmětu soft computing vedeném na vysokém učení technickém v brně na fakultě informatiky. 2020.