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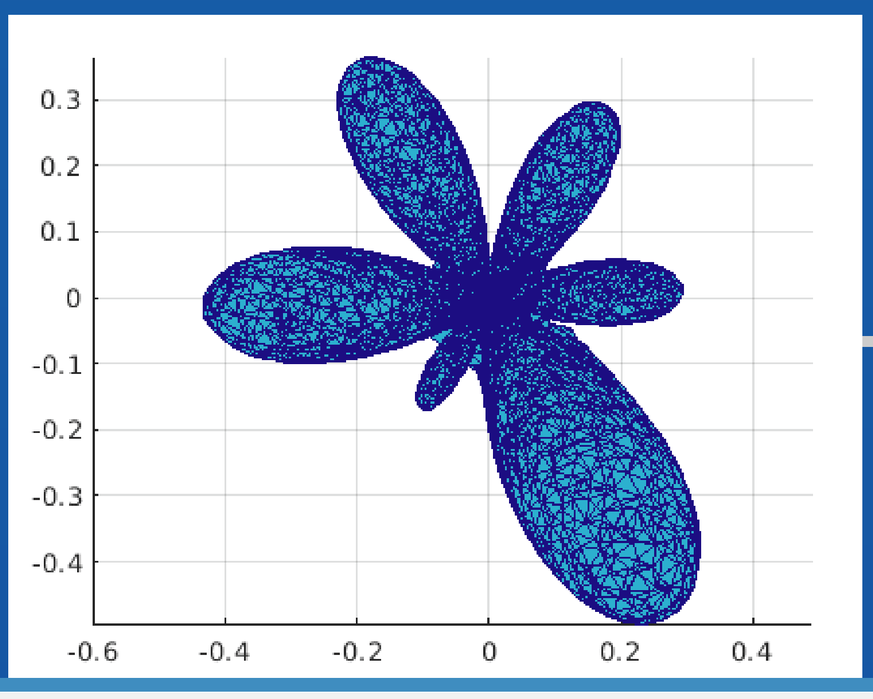
Importance sampling probability density functions represented with (hemi)spherical harmonics



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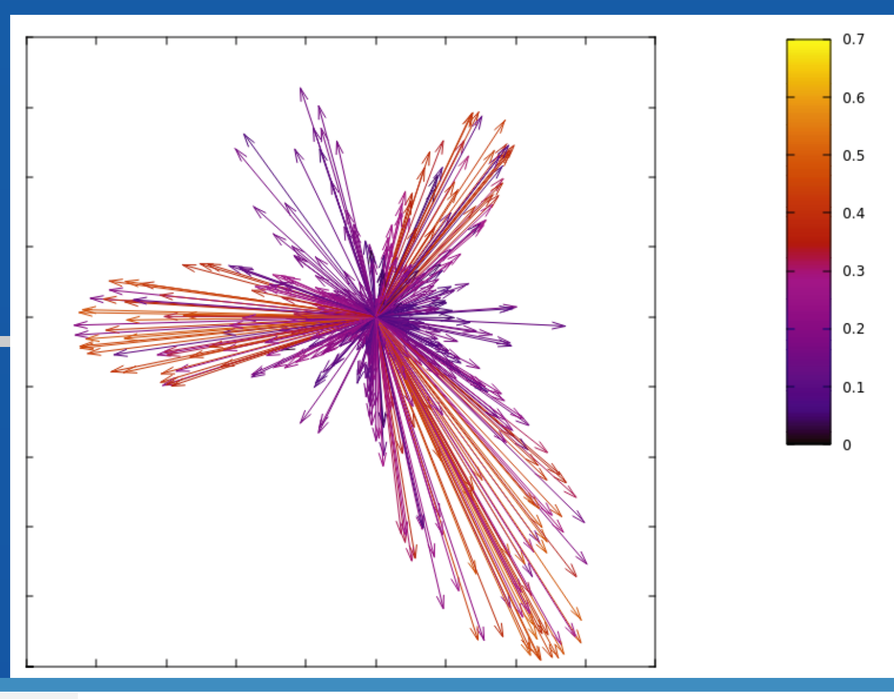
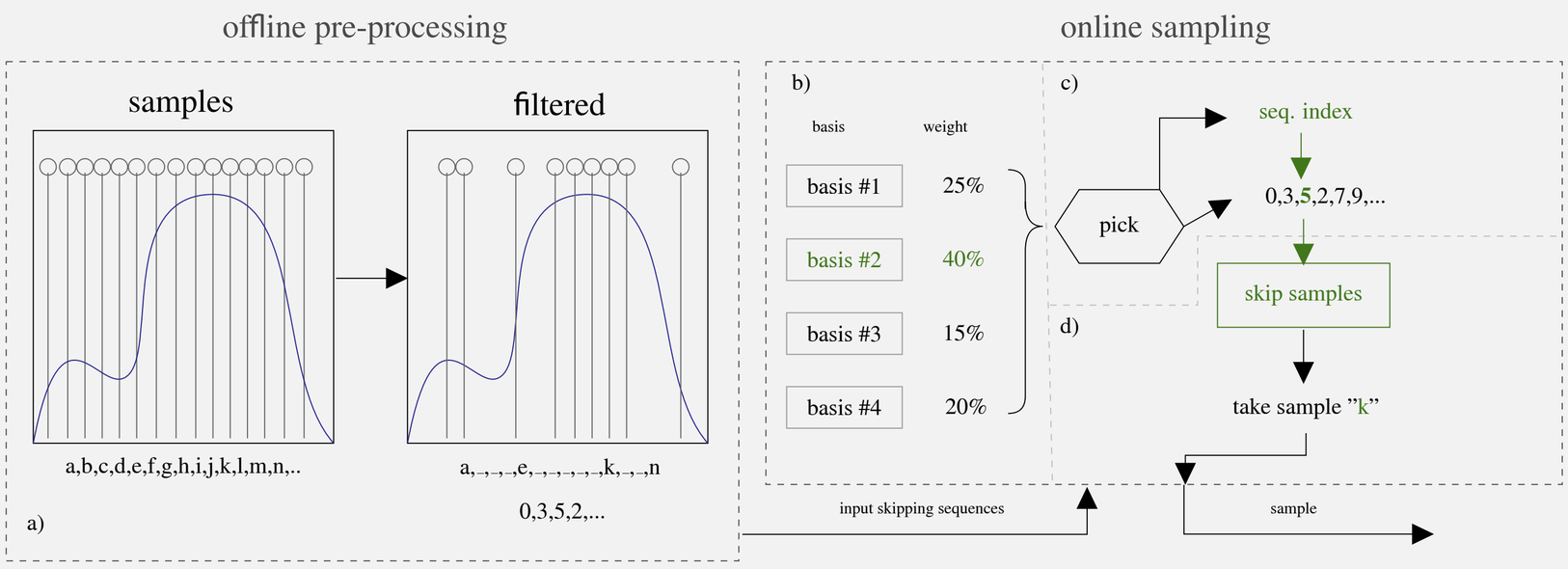
Importance sampling



Proposed algorithm



The constants l and m are integer values which represent the order l and the degree m of the basis functions. Formally, l is required to be a non-negative number and should satisfy the condition -l <= m <= l. Spherical harmonics are also orthonormal and rotation invariant

Approximation

Any real-valued spherical function can be approximated with SH by using a linear combination of spherical harmonics basis functions

$$f(\theta,\phi) \approx \sum_{l=0}^{N-1} \sum_{m=-l}^{l} f_l^m \cdot y_l^m(\theta,\phi)$$

where f are function coefficients computed by projecting a function onto each SH basis. These values can be estimated using various approaches, including Monte Carlo estimations

Hemispherical harmonics

As SH represents the whole spherical domain, in some cases, it might be useful to limit domain only to hemispherical. These functions are still based on associated Legendre polynomials with shifted domain using a linear transformation.

$$\widetilde{P}_l^m(\cos\theta) = P_l^m(2\cos\theta - 1) \text{ with } \theta \in [0, \frac{\pi}{2})$$

Comparison

In the contrary to existing methods, our approach does not require evaluation of any function integrals. The proposed approach generates over 3 million sample per seconds (while using single core) and does not decrease performance with increased size of SH basis.

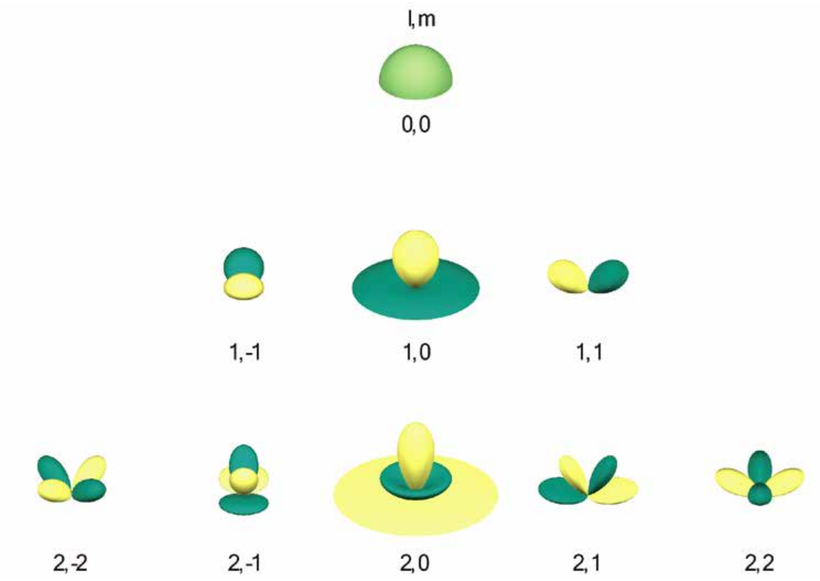
Spherical harmonics

$$y_l^m(\theta,\phi) = \begin{cases} \sqrt{2}K_l^m P_l^m(\cos\theta)\cos(m\phi) & m > 0 \\ \sqrt{2}K_l^m P_l^{-m}(\cos\theta)\sin(-m\phi) & m < 0 \\ K_l^0 P_l^0(\cos\theta) & m = 0 \end{cases}$$

Spherical harmonics represent spherical functions, as they are completely orthogonal basis on a sphere. Theyare based on associated Legendre polynomials

$$\begin{aligned} P_m^m(x) &= (-1)^m(2m-1)!!(1-x^2)^{m/2} \\ P_{m+1}^m(x) &= x(2m+1)P_m^m \\ (l-m)P_l^m(x) &= x(2l-1)P_{l-1}^m - (l+m-1)P_{l-2}^m \end{aligned}$$

SH basis functions



HSH basis functions