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Simulation of penitentes formation

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Abstract

This project aims to create a simulation of the process by which ice structures known as *nieve penitentes* are created. These structures look like tall cones and according to current research are shaped by scattering sun rays. The implementation is based on a mathematical model describing the process by M. D. Betterton, but several enhancements were made in order to bring this model closer to reality, most notably a fully three-dimensional approach to visible point detection, a key component of ablation rate computation. This simulation model was tested on both synthetic and real glacier surface data to show its features, not only proving its fundamental principles are correct but also noting its differences from the real-life process. Understanding the principle of how penitentes are being formed not only furthers our understanding of the world around us, but the ability to form them could help in reducing glacier melting.

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1. Introduction

The causes behind the formation of penitentes are not yet fully explored. The implementation of the simulation model and experimenting with it help with evaluating the knowledge we have about the phenomenon and show direction for the following research.

This project aims at implementing a simulation of process leading to the formation of ice structures called penitentes. The product should create results comparable to formations observable in nature. Comparisons can be both visual and numerical using suitable metrics.

The current most accepted theory of the formation of these structures is based on research done by a team around M. D. Betterton, who also designed a mathematical model of the formation of this phenomenon. This was previously implemented [1], however, it only worked with a surface defined as an univariate function. That is very limiting when comparing to formations in nature.

This implementation is using a mathematical model of the process by M. D. Betterton as a core, with stochastic raycasting added for approximation of computing the amount of light refracted into an area. This brings the model into a fully three-dimensional environment. Other improvements include variable solar intensity, which was constant before.

This solution is more rooted in the natural environment than its predecessors. It is able to form naturally looking formations of penitentes. Implementation with real data as input in mind makes it suitable for including extensions as research on this topic continues.

2. Principles of formation

Penitentes are tall ice cones that can be found in high altitudes about 4000 to 5200 meters above sea level, most known locations are in the Andes. The weather around is usually very sunny and cold. The core principle of their formation is differential ablation, which means some parts of a glacier melt faster than others. In this case, this is caused by Sun rays being scattered over the white snow and concentrating in depressions around the uneven surface. That leads to more melting energy inside these depressions than the rest of the surface, as depicted in Figure 2 [2]. Some researchers suggest that creating penitentes artificially could be a way of slowing down the melting of that glacier[3].

2.1 Mathematical model

Model by M. D. Betterton considers sunlight as a primary source of heat in the model. The model is described as a differential equation of change of height of the surface $h(x)$ at point x. The surface reflects sunrays unitarily in all directions, therefore the amount of heat reflected into one point is dependent on the points from which the light can reflect.

3. Implementation

This implementation is done in C++ language, using OpenMP library for parallelization and *plycpp* library from GitHub[1] for handling *.ply* format files, that are used as both input and output.

The core of the problem is the computation of a differential equation for the change of surface height in one point over time including an integral over all points of the surface visible from the examined point. The exact equation is in Figure 3 [4]. This is solved using Monte Carlo approximation method and ray casting. A large number of vectors uniformly distributed in a sphere around the examined point is sent into the scene and their hits on the surface are evaluated. From this, the visible points are determined and through normalization, an approximation of the integral over the visible surface is computed.

The evaluation of detected hits consists of computing hit point of the ray and plane, verifying whether this point is in the face of the mesh that defines the plane by its vertices and finding the closest of these hit points using a parametric representation of the ray, as Figure 4 depicts. Finally, verification of whether the hit point and examined point are facing each other is done using the dot product of normals in their location on the surface.

The program passes all vertices of the mesh and computes the height difference in the time step. Due to the way how *.ply* format is being saved, multiple vertices with the same location, but different normals are being evaluated at the same time. This is solved by having a hash table with all the vertices sorted by location. Parallelization is done on the ray level. Multiple rays sent from one examined point are being evaluated at the same time.

Unlike the original mathematical model, this implementation uses variable solar intensity. This was chosen to address huge decrements in the surface at steep sides, where the constant intensity was previously assumed. Change of intensity is based on the dot product of the surface normal and vertical line of sun rays.

4. Experiments

The purpose of experiments on this project is to find out the properties of the model implemented here. The experiments done on synthetic data were supposed to show model does create structures similar to penitentes found in nature. The input surface was made slightly rough using noise, and during multiple time steps in the simulation, the structures were formed. The render of the result can be seen in Figure 6.

The results were compared to results that the previous implementation [1] made with equal input data. Both variants produce similar results, but this implementation tends to form more precisely, thanks to its fully 3D approach.

Other experiments focused on applying the model on scan of a natural penitentes field[5]. These meshes however contain structures like hollows and overhangs, for which the model might not react properly as it is defined around a surface described by a mathematical function.

The results are seen in Figure 7 and further investigation of these points showed that overly huge increments were caused on very steep surfaces, which in reality would catch a very small amount of sunlight, however, the model counts them as fully lighted due to the solar intensity being considered constant.

To improve results the variability of solar intensity based on the slope of the surface was developed, but shading is still partly neglected and could be one of the points of continued work on this project. Result of the modified model can be seen in Figure 8.

5. Conclusions

This project focused on implementing the mathematical model for the formation of penitentes by M. D. Betterton with expansion in a fully three-dimensional approach in visible points search.

The model shows its ability to form structures similar to natural penitentes from synthetic input surfaces. Its reaction to the real-life-based surface is caused by the approach selected in the mathematical model, which left several situations undefined. Further work on this project could lead to the calculation of Sun position and shades.

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References

[1] Ondřej Čech. *Simulace sněhových a ledových struktur*. Bakalářská práce. Vysoké učení tech-

[1] https://github.com/rbregier/plycpp

nické v Brně, Fakulta informačních technologií. Vedoucí práce doc. Ing. Martin Čadík, Ph.D., 2020.

[2] Louis Lliboitry. *The Origin of Penitents*. Journal of Glaciology, s. 2:331–338, 1954.

[3] Catherine Brahic. Spiky glaciers are slower to melt. https://www.newscientist.com/article/dn11309-spiky-glaciers-are-slower-to-melt/.

[4] Meredith D. Betterton. *Formation of Structure in Snowfields:Penitentes, Suncups, and Dirt Cones*. Physical Review E, vol 63, 2001.

[5] Lindsey I. Nicholson, Michał Petlicki, Ben Partan, and Shelley MacDonell. *3-D surface properties of glacier penitentes over an ablation season, measured using a Microsoft Xbox Kinect*. The Cryosphere (10):1897–1913, 2016.