Simulation of penitentes formation

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Penitentes

Figure 1: Penitentes field in nature

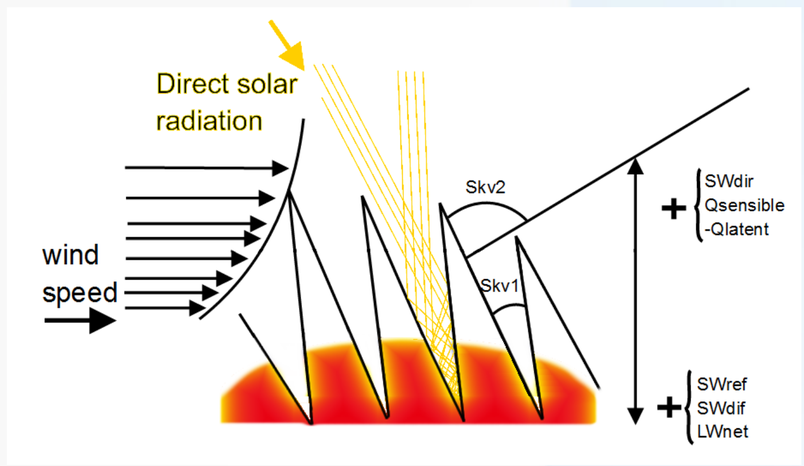

Figure 2: Illustration of the principle of how penitentes are formed

Mathematical model

Albedo (constant)

Solar intensity: how much energy is given to the surface unit

$$\frac{\delta h}{\delta t} = \frac{-\alpha I}{L}\left(\frac{1}{\pi}\int_{line\ of\ sight}\frac{\Delta h - h'(x)\Delta x}{\Delta h^2 + \Delta x^2}dx\right)$$

The heat needed to melt volume unit of ice (constant)

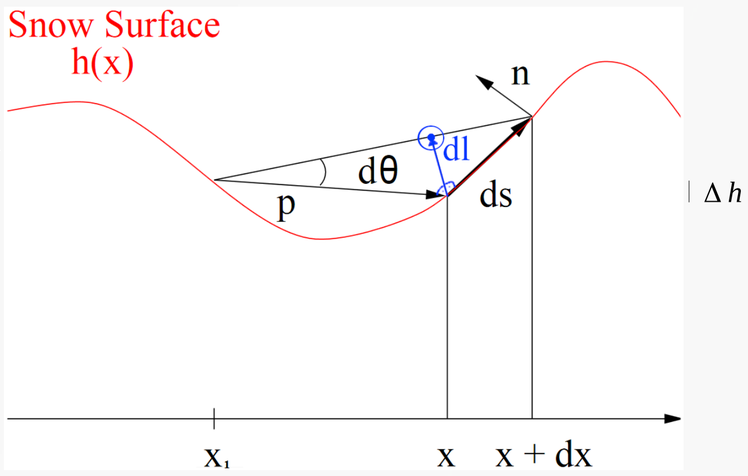

Figure 3: differential equation of the mathematical model

Implementation

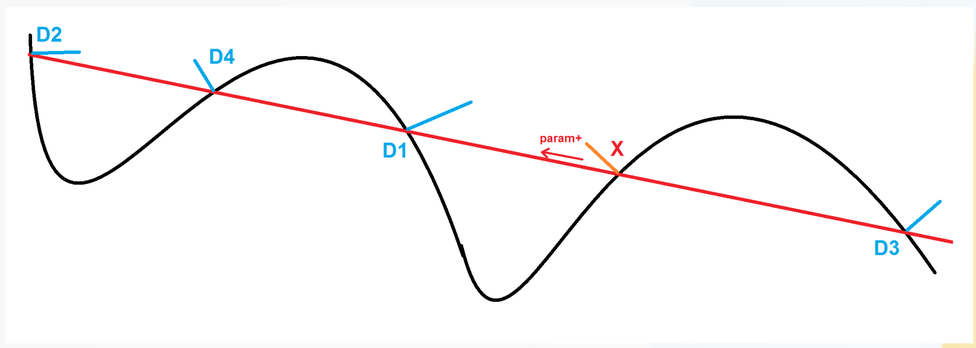

Figure 4: Hit point evaluation using the parametric line. Ray sent from X identifies hit points D1 to D4. D3 is rejected because the parameter is negative there, D4 is rejected because it is not facing X, D1 is decided as visible because it has a lower parameter value than D2.

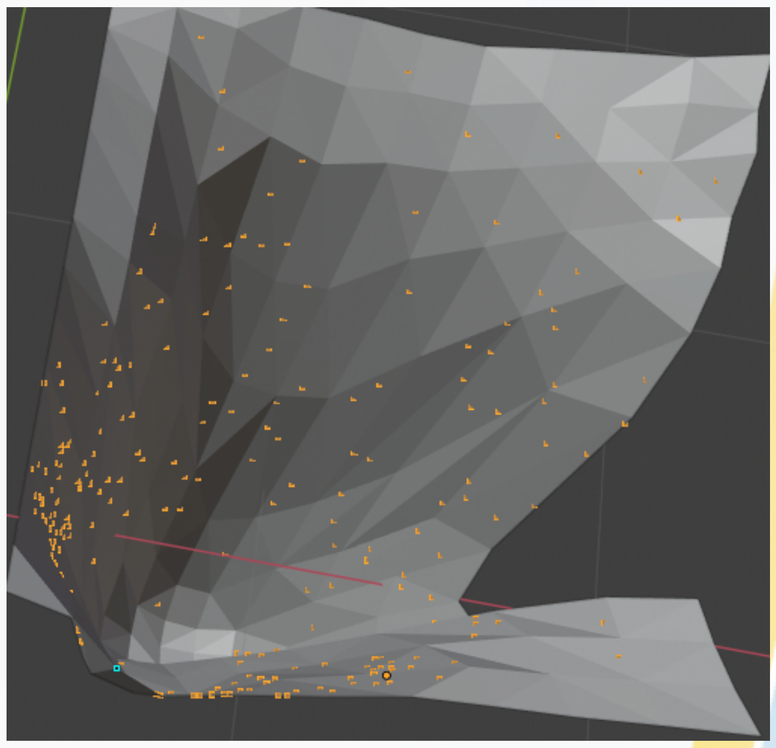

Figure 5: Visible point from cyan examined point on the testing surface used for Monte Carlo integration in the differential equation of the model

Results – real data

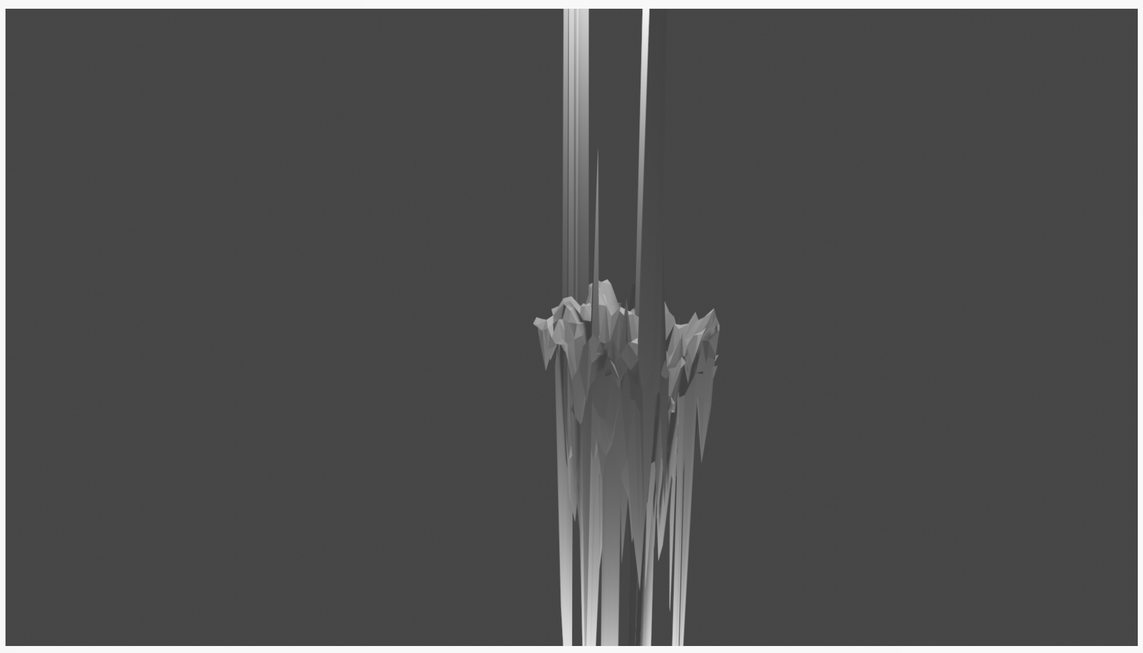

Figure 7: Results of the basic model on real input data. Worth noticing are the spikes in both directions. The points causing this have an overwhelming number of visible points on steep surfaces. These points would refract only a fraction of sunlight, but the model considers them fully lighted due to constant solar intensity.

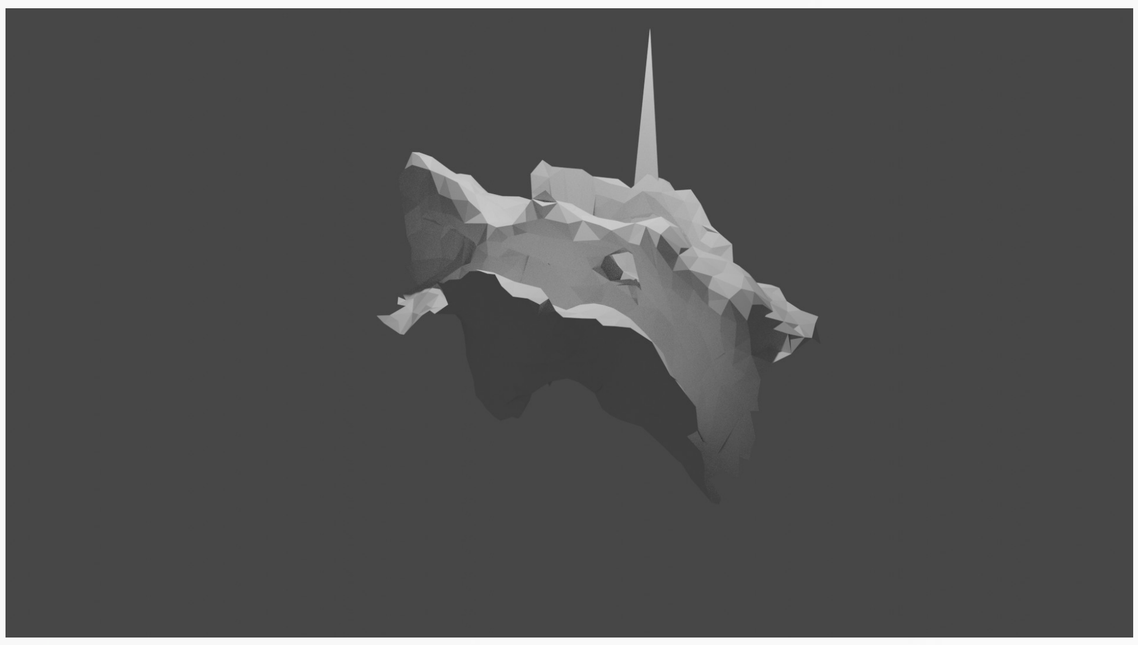

Figure 8: Results of model modified with variable solar intensity. While some error spikes remained, much of their creation was either eliminated or very reduced.

Results – synthetic data

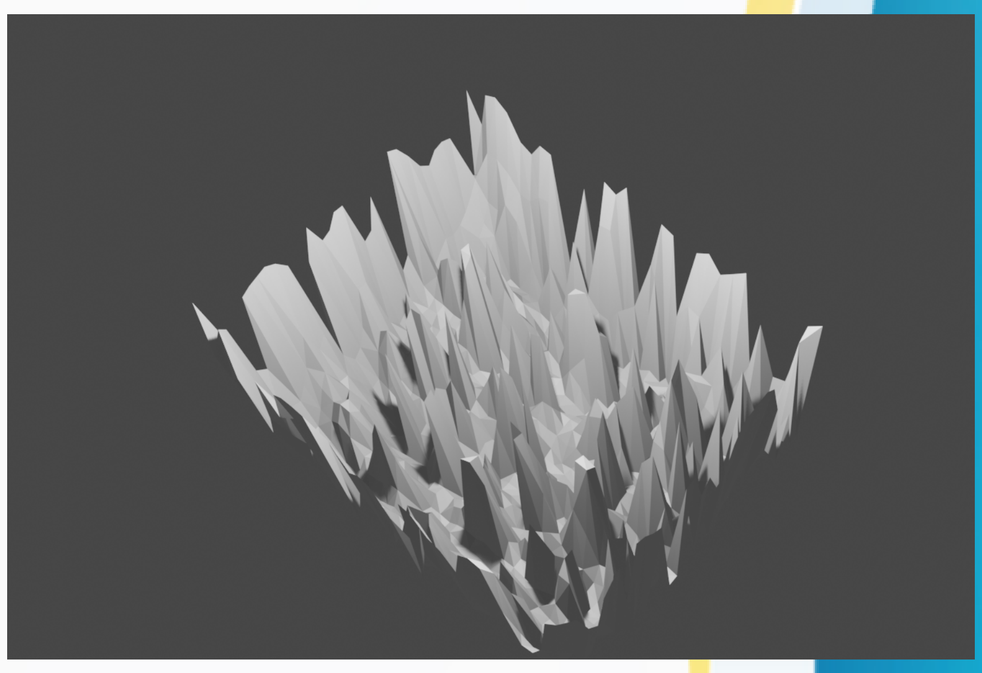

Figure 6: Results of the model on synthetic data. There are clearly visible spikes formed during the run of the simulation that are similar to natural penitentes