BRNO UNIVERSITY OF TECHNOLOGY FACULTY OF INFORMATION TECHNOLOGY

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Efficient Reduction of Finite Automata

Veronika Molnárová - xmolna08@stud.fit.vutbr.cz

Supervisor: Ing. Ondřej Lengál, Ph.D

Objective

- find smaller automaton defining the same language
- DFAs may be exponential in size compared to NFAs

→ **we want to reduce NFAs directly**

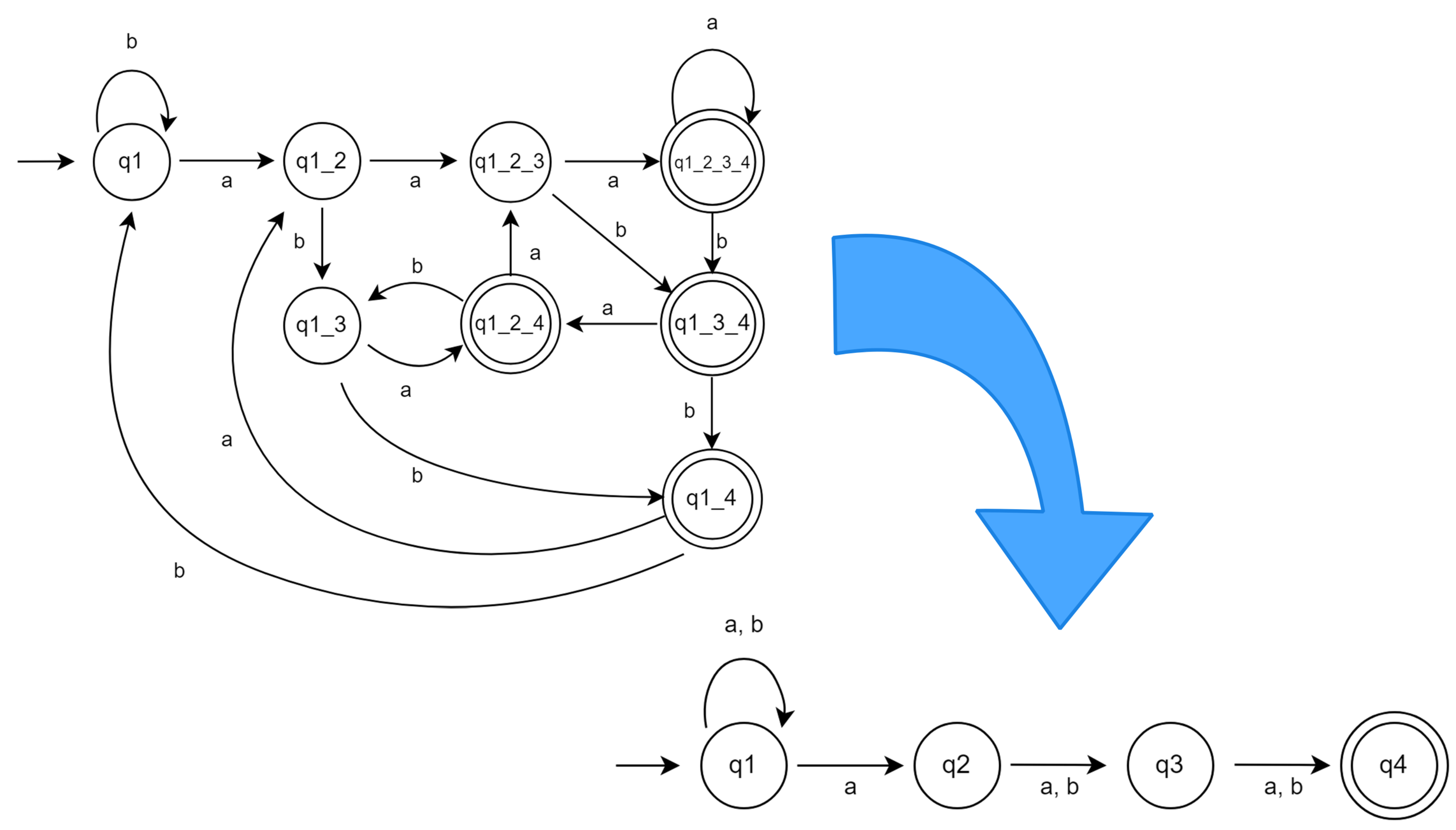

Figure 1: Comparison of DFA (on the top) and NFA (on the bottom) defining the same language. We can see the clear advantage of NFAs for their smaller size.

Reduction utilizing SAT and QBF solvers

$-T_{1a1} \vee -T_{1a2}$ $-T_{1b1} \vee -T_{1b2}$ $-T_{2a1} \vee -T_{2a2}$ $-T_{2b1} \vee -T_{2b2}$	$T_{1a1} \vee T_{1a2}$ $T_{1b1} \vee T_{1b2}$ $T_{2a1} \vee T_{2a2}$ $T_{2b1} \vee T_{2b2}$
$T_{1a1} \wedge T_{1b1} \wedge F_1$ $T_{1a1} \wedge T_{1b2} \wedge F_2$ $T_{1a2} \wedge T_{2b1} \wedge F_1$ $T_{1a2} \wedge T_{2b2} \wedge F_2$	$-T_{1b1} \wedge -T_{1a1} \wedge -F_1$ $-T_{1b1} \wedge -T_{1a2} \wedge -F_2$ $-T_{1b2} \wedge -T_{2a1} \wedge -F_1$ $-T_{1b2} \wedge -T_{2a2} \wedge -F_2$

Figure 3: Clauses representing a DFA with 2 states, 2 symbols of the alphabet, accepting word 'ab' (blue) and rejceting word 'ba' (green) for SAT solver. The clauses with yellow and orange background are clauses for determinism and completeness. Using only blue and green clauses including also variables for initial states we can define an NFA.

$Q_0 \vee I_1$ $-Q_0 \vee I_2$ $Q_2 \vee F_1$ $-Q_2 \vee F_2$ $-Q_3 \wedge -I_1$ $Q_3 \wedge -I_2$ $-Q_5 \wedge -F_1$ $Q_5 \wedge -F_2$	$Q_0 \vee Q_1 \vee T_{1a1}$ $Q_0 \vee -Q_1 \vee T_{1a2}$ $-Q_0 \vee Q_1 \vee T_{2a1}$ $-Q_0 \vee -Q_1 \vee T_{2a2}$ $Q_1 \vee Q_2 \vee T_{1b1}$ $Q_1 \vee -Q_2 \vee T_{1b2}$ $-Q_1 \vee Q_2 \vee T_{2b1}$ $-Q_1 \vee -Q_2 \vee T_{2b2}$	$-Q_3 \wedge -Q_4 \wedge -T_{1b1}$ $-Q_3 \wedge Q_4 \wedge -T_{1b2}$ $Q_3 \wedge -Q_4 \wedge -T_{2b1}$ $Q_3 \wedge Q_4 \wedge -T_{2b2}$ $-Q_4 \wedge -Q_5 \wedge -T_{1a1}$ $-Q_4 \wedge Q_5 \wedge -T_{1a2}$ $Q_4 \wedge -Q_5 \wedge -T_{2a1}$ $Q_4 \wedge Q_5 \wedge -T_{2a2}$

Figure 4: Clauses representing an NFA for QBF solver with the same parameters as the automaton above. Uses quantified variables Q0-Q5 representing states of a path in the automaton. Yellow and orange are clauses for initial and final states, blue accepting and green rejecting clauses.

Results of implemented algorithms

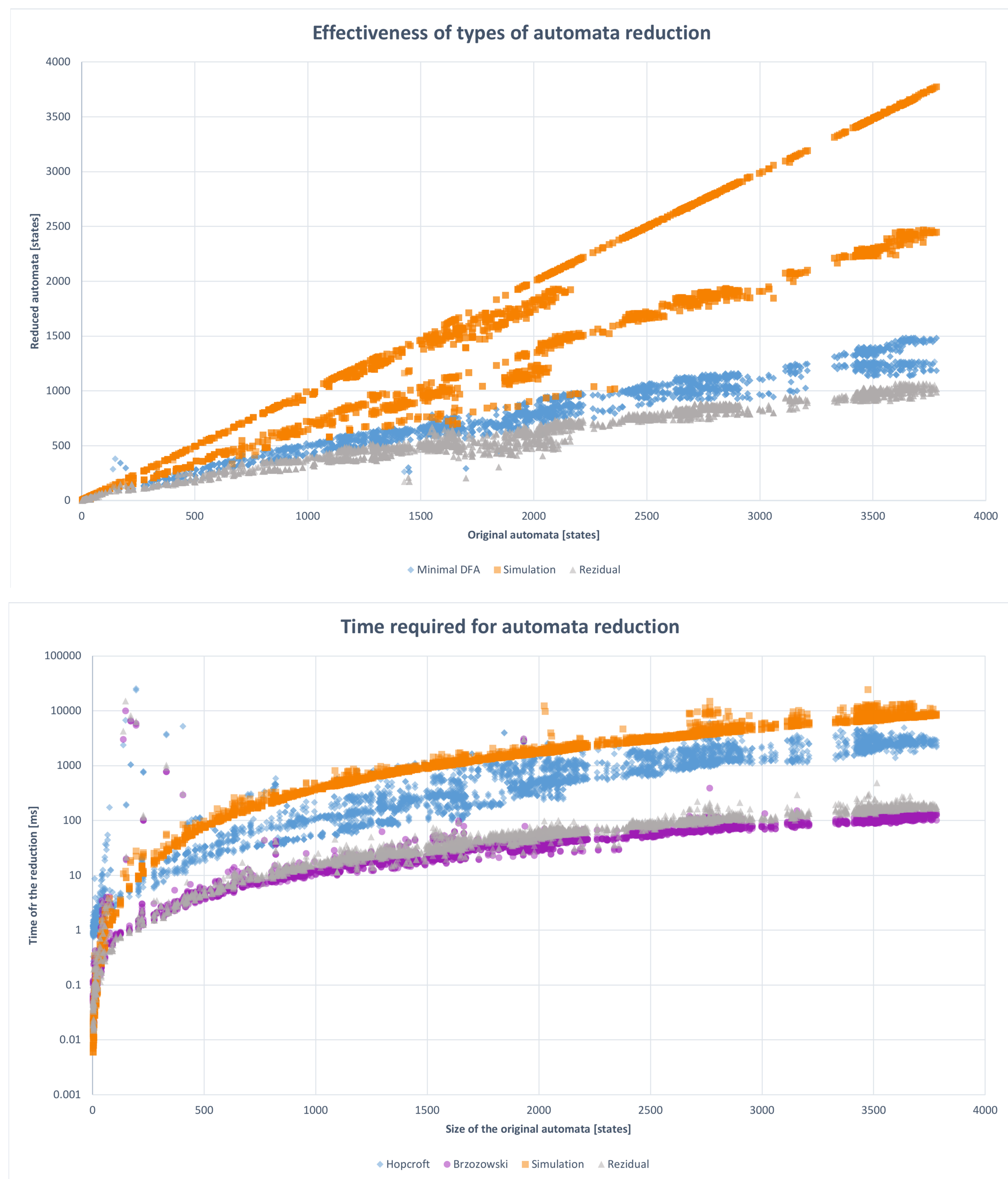

Figure 2: Graphs showing the results of implemented reduction algorithms, which are the minimization of DFA (using Hopcroft's and Brzozowski's algorithms), the reduction based on a relation of simulation, and the reduction utilizing residual automata.

SAT and QBF result predictions

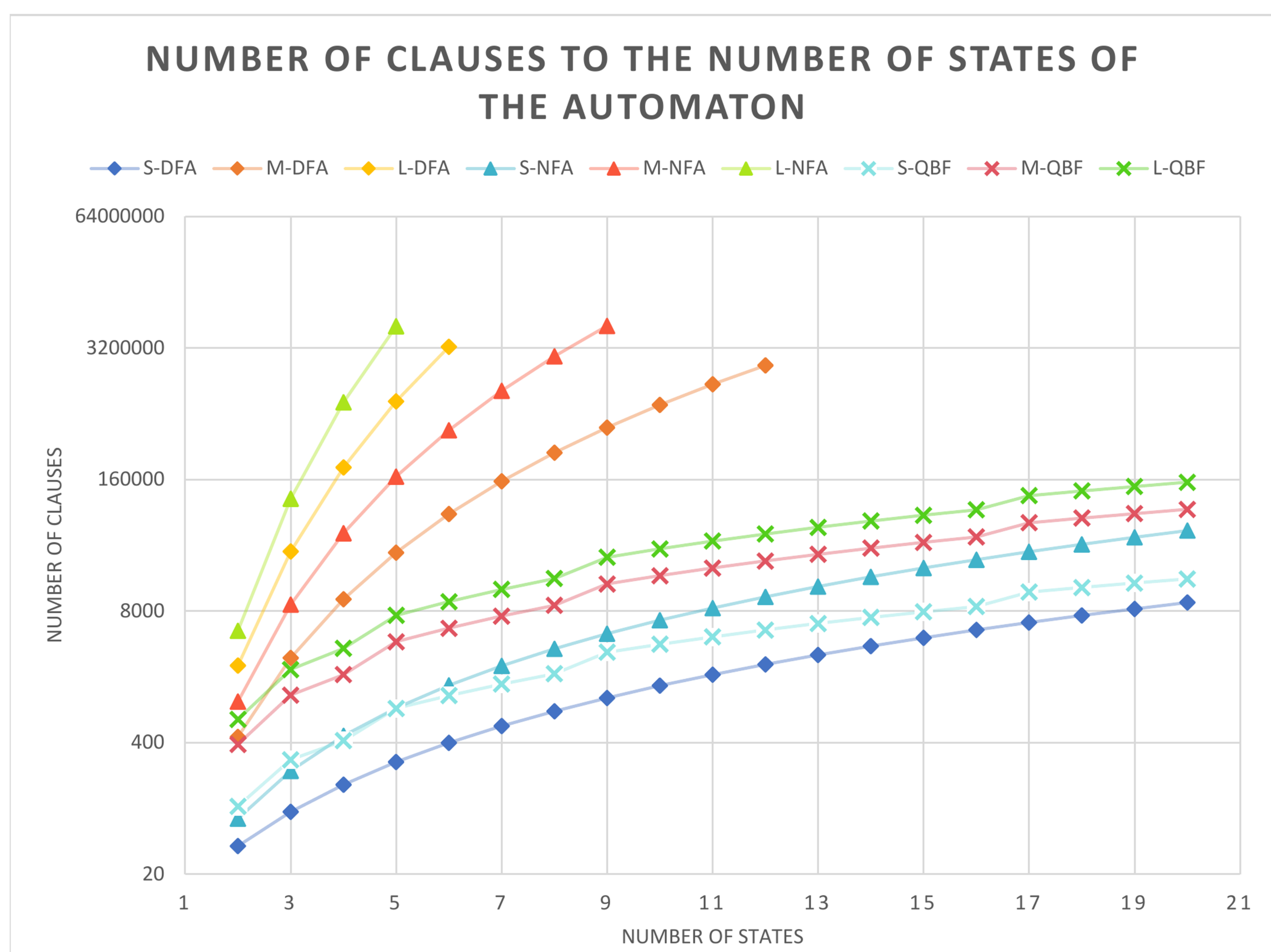

Figure 5: Graph showing the number of clauses generated for each type of automaton: DFA(SAT), NFA(SAT), QBF. An input is the number of states and an example set of input words with 3 sizes (S, M, L). The number of symbols of the automaton was set to a constant of 2. The SAT solvers require exponential number of clauses based on the number of states and the length of the word. For QBF it is polynomial number of clauses.